Advanced Instructional School
(Sponsored by NBHM)

Algebraic Combinatorics

Indian Statistical Institute
Bangalore Centre

June 24 - July 13, 2013

Organizers : N.S.N. Sastry, ISI, Bangalore and
Sudhir R. Ghorpade, IIT Bombay

Faculty

Bhaskar Bagchi	ISI, Bangalore
Amritanshu Prasad	IMSc., Chennai
Sudhir Ghorpade	IIT Bombay
Sharad S. Sane	IIT Bombay
N.S.N. Sastry	ISI, Bangalore
Murali K. Srinivasan	IIT Bombay

Unity in Mathematics Lectures:

N. M. Singhi	TIFR, Mumbai
R. Balasubramanian	IMSc., Chennai

Tutorials: Binod Kumar Sahoo, Samrith Ram, Satraj Ul Hassan (Tentative list)

A brief description of the School: This school is addressed to research students and young researchers in algebraic combinatorics and applications. In this programme, we plan comprehensive lectures on several major topics in combinatorics, augmented by tutorials and additional lectures related to the themes of the main lectures. Some of the topics would be : Witt designs and their automorphisms, finite incidence geometries, combinatorics of Leech lattice, combinatorics of Young tableaux and symmetric polynomials, linear error correcting codes and some of their interrelations with algebraic geometry, combinatorial applications of linear algebra.

Contact details:

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Combinatorics of Young Tableaux
Amritanshu Prasad

This course will cover the basic combinatorial properties of Young tableaux, focusing mainly on the Robinson-Schensted algorithm. Constructions of this algorithm, its basic properties and its consequences in the theory of symmetric functions will be discussed.

Sources are to include:

1. The Art of Computer Programming, Volume 3, by Donald E. Knuth (Chapter 5).
2. The Robinson-Schensted and Schuetzenberger algorithms, an elementary approach, by Marc. A. A. van Leeuwen.
3. Enumerative Combinatorics, Volume 2, by Richard P. Stanley (Chapter 7).

Prerequisites: A standard course in linear algebra, a familiarity with finite groups and their actions on sets.

Witt designs and some related mathematics
Sharad S. Sane

After covering the necessary background from design theory including symmetric designs, projective designs and the design extension problem, the set of lectures will go on to construct Witt designs, that is, the Steiner systems on 12 and 24 points respectively and will also discuss the automorphism groups of these structures which are Mathieu groups with various degrees. This will also enable us to construct the Higman-Sims graph and its automorphism group, the Higman-Sims group which was historically the first example of a sporadic simple group (almost hundred years after the discovery of the first five sporadic simple groups called the Mathieu groups).

Combinatorial applications of linear algebra
Murali K. Srinivasan

Using elementary linear algebra and finite group actions we shall present a selection of attractive results in enumerative/algebraic combinatorics. Topics include, but are not restricted to, the following:

(i) Counting spanning trees in graphs.

(ii) Counting Hasse walks with applications to counting standard Young tableaux.

(iii) Unimodality of the q-binomial coefficients.

(iv) Eigenvalues of the classical Bose-Mesner algebras.

(v) Recent results on explicit block diagonalization.

To get an idea of some of the contents and level of these lectures kindly look at:

1. Topics in Algebraic Combinatorics, by R. P. Stanley (available: `http://www-math.mit.edu/~rstan/algcomb.pdf`).

2. 33 Miniatures: Mathematical and Algorithmic Applications of Linear Algebra, by J. Matousek, AMS (2010).

There is considerable overlap but also significant differences from these two sources so we will provide lecture notes.

Aspects of Coding Theory
Sudhir R. Ghorpade

Topics to be discussed in these lectures include the following:

1. A brief introduction to linear codes, basic notions and results including elementary bounds and groups of automorphisms of codes. Outline of an alternative geometric approach to codes via projective systems.

2. Basic examples. A particular emphasis will be on generalized Reed-Muller codes and an explicit determination of various objects associated them, including basic parameters, duals, and automorphism groups.

3. The role of Grassmann varieties in coding theory, linear codes associated to certain higher dimensional algebraic varieties.

Sources will include parts of the 2-volume *Handbook of Coding Theory* and some relevant research papers.

From the Witt design $S(5, 8., 24)$ to the Leech lattice
Bhaskar Bagchi

In these lectures, we shall discuss the Leech lattice, its existence and uniqueness, beginning with the uniqueness of $S(5, 8, 24)$. Time permitting, we may also relate it to the unique even interval lattice in 26-dimensional Minkowski space-time.

Generalized polygons and polar spaces
N.S.N. Sastry

In these lectures, we shall discuss generalized polygons and polar spaces; ovoids and spreads in them. We emphasize their basic role in the structure of geometric objects related to the structure of finite simple and algebraic groups.

In greater detail, the following topics will be covered. Finite generalized polygons, Feit-Highman Theorem; Polar spaces, Description of all polar spaces according to Tits-Veldkempf classification, Ovoids and Spreads in polar spaces. Some relations to associated codes.

References: 1. S. Payne and J.A. Thas, Finite generalized polygons, Pitman, Boston, 1984. 2. E. Moorehouse, Incidence geometry
(weblink: `http://www.uwyo.edu/moorhouse/handouts/incidence_geometry.pdf`)