Lecture (Date 22/11/16) (Parasar Mohanty): CONVOLUTION .

Definition and basic properties of Fourier transform of functions in $L^2(\mathbb{R})$ and functions in other spaces is discuussed.

Lecture (Date 21/11/16) (Parasar Mohanty): CONVOLUTION .

Let us have notations $\mathbb{T}$ = unit circle in complex plane $\mathbb{C}$. $\mathbb{R} = (-\infty, \infty)$. Let $< X, M, \mu >$ be a measure space.$L^p(X) = \left\{ f : X \longrightarrow \mathbb{C} \ : \ \int_X |f(x)|^p dx < \infty \right\}$.
$L^\infty = \left\{ f : X \longrightarrow \mathbb{C} \ : \ \sup_{x \in X} |f(x)| < \infty \right\}$.
Following problems are discussed:
Problem 1.: Show that, $L^\infty(\mathbb{T}) \subset$ .... $\subset L^2(\mathbb{T}) \subset L^1(\mathbb{T})$ .While we have niether $L^1(\mathbb{R}) \subseteq L^2(\mathbb{R})$ nor $L^2(\mathbb{R}) \subseteq L^1(\mathbb{R})$ We have $f(x) = \frac{1}{x}\chi_{[1,\infty)} \in L^2(\mathbb{R}) \backslash L^1(\mathbb{R})$ and $g(x) = \frac{1}{\sqrt{x}}\chi_{[0,1]} \in L^1(\mathbb{R}) \backslash L^2(\mathbb{R})$.
Now $\ell_1(\mathbb{Z}) \subseteq \ell_2(\mathbb{Z}) \subseteq \ell_3(\mathbb{Z}) \subseteq$ .....$\ell_\infty(\mathbb{Z})$.
Problem 2.:Show that , $C_c(\mathbb{R})$ set of all continuous functions with compact support is dense in $L^1(\mathbb{R})$.
Now for $f, g \in L^1(\mathbb{R})$ we can define convolution $f * g(x) = \int_{\mathbb{R}} f(y)g(x - y)dy$.
Problem 3: prove that $||f * g||_1 \leq ||f||_1||g||_1$. Hence $f, g \in L^1(\mathbb{R})$ implies $f * g \in L^1(\mathbb{R})$.
Note:If there exist $h \in L^1(\mathbb{R})$ such that $f*h = h$ .Then $\widehat{f * h} = \widehat{f}\widehat{h}$ which implies $\widehat{h} = 1$.Which is not possible.

Lecture (Date 23/11/16) (Parasar Mohanty): APPROXIMATE IDENTITY WITH RESPECT TO CONVOLUTION.

He Discused following results:
Result 1: If $f \in L^1(\mathbb{R})$ then (by using of Riemann Lebesgue Lemma) Fourier transform of f ,that is $\widehat{f} \in C_o(\mathbb{R})$.
Now $f \in L^p(\mathbb{R}), g \in L^q(\mathbb{R})$ then by Holder's inequality convolution $f * g \in L^\infty(\mathbb{R})$ and continuous on $\mathbb{R}$ also. Here $\frac{1}{p} + \frac{1}{q} = 1$.
In general we have discussed following problems:
Problem (1): If $f, g \in C_c(\mathbb{R})$ then $\tau_x(f * g) = \tau_x(f) * g = f * \tau_x(g)$.
Problem (2): Let $T : L^p(\mathbb{R}) \longrightarrow L^p(\mathbb{R})$ be bounded linear operator and translation invariant that is $T(\tau_x(f) = \tau_x(T(f))$ for all $f \in L^p(\mathbb{R})$. Then $T(f * g) = T(f) * g = f * T(g)$ for all $f, g \in L^p(\mathbb{R})$.
Problem (3): If $f_k \in L^p(\mathbb{R})$ then prove that $(\int \sum_{k=1}^{n} |f_k|^p)^{1/p} \leq \sum_{k=1}^{n} (\int |f_k|^p)^{1/p}$.
Problem (4).: If $f \in L^p(\mathbb{R})$ then prove that $(\int_{\mathbb{R}} \int_{\mathbb{R}} |f_k|^p)^{1/p} \leq \int_{\mathbb{R}} (\int_{\mathbb{R}} |f_k|^p)^{1/p}$.

Problem (5).: If $f \in L^1(\mathbb{R}), g \in L^p(\mathbb{R})$ then prove that $f * g^p(\mathbb{R})$.
Problem (6).: Let $1 < p < 2$ and $f \in L^p(\mathbb{R})$ then show that $f = f\chi_A + f\chi_{A^c}$ .Here $A = \{x \ : \ |f(x)| > 1\}$ By using this we have defined Fourier tranform of functions in $L^p(\mathbb{R})$ for $1 < p < 2$.
Also we have discuussed about Hilbert transform on these sapces.
We introduced the definition of approximate identity.
A famlily of functions $\{phi_n\}_{n=1}^{\infty}$ satisfying following
1. $\int_{\mathbb{R}} \phi_n(x)dx = 1$.
2. $\int_{\mathbb{R}} |\phi_n(x)|dx \leq K \quad \forall \ n = 1, 2, ...$
3.$\int_{|x|>\delta} |\phi_n(x)|dx0$ as $n \longrightarrow \infty$.
For this approximate identity $f * \phi_n \longrightarrow f$ in L^1 norm as well as pointwise.

Lecture (Date 24/11/16) (Parasar Mohanty): DICRETE CONVOLUTION .

We have discussed following problems:
Problem 1: Let $a = (a(n))_{n=1}^{\infty}, b = (b(n))_{n=1}^{\infty} \in \ell_1(\mathbb{Z})$.We define $a * b(n) = \sum_{m \in \mathbb{Z}} a(m)b(n-m)$
Show that $a * b \in \ell_1(\mathbb{Z})$.
Problem 2: Adress the existence of identity in $\ell_1(\mathbb{Z})$ with respect to above definition of convolution.

Lecture (Date 25/11/16) (Parasar Mohanty):FUNDAMENTAL THEOREM OF CALCULUS ON $\mathbb{R}^n$

. Definition of Hardy Littlewood maximal function and examples are discusssed.
Statement and proof of Vitali covering Lemma is discussed.
Using this Fundamental theorem of Calculus for on regions in $mathbbR^n$ is discussed.

Lecture (Date 26/11/16) (Parasar Mohanty): FOURIER ANALYSIS ON FINITE GROUPS

He has given following definition:
Let G be finite abelian group. and $\mathbb{T}$ be unit circle in complex plane $\mathbb{C}$. Let $\phi : G \longrightarrow \mathbb{T}$ be a such that $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in G$ (that is group homomorphism).ϕ is called as character of G.Let us denote $\widehat{G}$ be set of all characters of G.
Note that $\widehat{\mathbb{Z}_N} = \mathbb{Z}_N$. $\widehat{\mathbb{T}} = \mathbb{Z}$.
He has proved following Lemmas:
Lemma(1):If $\gamma : G \longrightarrow \mathbb{C} - \{0\}$ satisfying $\gamma(xy) = \gamma(x)\gamma(y)$ then $\gamma \in \widehat{G}$.
Lemma(2):Let X is vector space of all functions from G to $\mathbb{C}$ with inner product $< f, g >= \frac{1}{|G|} \sum_{x \in G} f(x)\overline{g(x)}$ .Then $\widehat{G}$ forms orthonormal system for X.
Lemma(3):Let $\{T_1, T_2, ...\}$ be family of diagnolisable linear operator on a finite dimensional vetcor space V.If $T_iT_j = T_jT_i$ then there exist basis B of V such that matrix of each T_i with

respect to B is diagonal.
Using these lemmas he proved following main theorem
Theroem:$\widehat{G}$ forms orhonormal basis for X.
One application:
Let $\pi = \{z(0), z(1), ..., z(k-1)\}$ be a closed polygon in the plane and $Dz(j) = \frac{1}{2}(z(j-1) + z(j) = \frac{1}{2}(\delta_1 + \delta_0) * z(j)$ be midpoint of side $z(j-1)$ to $z(j)$.Here $*$ is convolution.
Now $\pi' = \{Dz(0), Dz(1), ..., Dz(k-1)\}$ is derived polygon of π and $\pi'', \pi^3, ..., \pi^k$ be derived polygons sucessively.Then π^m approches to centroid of π namely $p = \frac{1}{k}(z(0) + z(1) + ... + z(k-1)$ as m tend to infinity.