REPORT: IST IN FOURIER ANALYSIS, BP, PUNE, NOVEMBER 2016

1. Lecture 1: The Weierstrass approximation theorem

In this lecture, we exploit a family of good kernels to prove the Weierstrass approximation theorem.

2. Lectures 2 and 3: Applications to some partial differential equations

The first PDE we discussed is the time-dependent heat equation on the real line $u_t = u_{xx}$ subject to the boundary condition $u(x, 0) = f(x)$. In particular, we discuss existence and uniqueness of its solution under modest assumptions. The second PDE we discussed is The steady-state heat equation in the upper half-plane. The solutions for these PDE's are obtained by convoluting Heat and Poisson kernels with the boundary function f.

3. Lecture 4: The Poisson summation formula

In this lecture, we established the Poisson summation formula, and discuss applications to theta and zeta functions.

4. Lectures 5 and 6: Paley-Wiener theorem

We discuss the problem of holomorphic extension to a strip or complex plane. We show that it is related to the growth of the Fourier transform of the function under condition. In particular, we prove Paley-Wiener Theorem. Its proof relies on a maximum modulus principle for a sector. We also prove a version of uncertainty principle stating that a function and its Fourier transform can not have compact simultaneously unless it is zero.

References

[1] E. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press, Princeton and Oxford, 2002.

[2] E. Stein and R. Shakarchi, *Fourier Analysis*, Princeton University Press, Princeton and Oxford, 2002.