Instructional School for Teachers, Fourier Analysis
November 2016, Bhaskaracharya Pratishthan, Pune
Lectures by Dr. V. M. Sholapurkar

Lecture 1: 14-11-2016
Consider 2π periodic functions. Definition of Fourier series, Definition of convolution
N^{th} Dirichlet kernel: $D_N(x) = \sum_{n=-N}^{N} e^{inx}$
Properties of Dirichlet kernel, Definition of Good kernels

Lecture 2: 15-11-2016
Proof of the following theorem:
Theorem: If $\{K_n\}$ is a family of good kernels and f is 2π periodic integrable function which is continuous at x, then

$$\lim_{n\to\infty}(f * K_n)(x) = f(x).$$

Notion of Cesaro summability
Fejer Kernel: $F_N(x) = \dfrac{D_0(x) + \cdots + D_{N-1}(x)}{N}$
This is example of a family of good kernels.
Notion of Abel summability

Lecture 3: 16-11-2016
Statement of Uniqueness problem
Proof of trignometric polynomials are dense in the set of continuous functions on unit circle.
Statement of Dirichlet problem
Poisson kernel: $P_r(\theta) = \sum_{n=-\infty}^{\infty} r^{|n|}e^{in\theta}$
This is example of a family of good kernels.

Lecture 4: 17-11-2016
Dirichlet kernel is not a good kernel.
Applications:
1) Isoperimetric Inequality
2) Parseval's Identity

Lecture 5,6: 18-11-2016

Fourier Transform on $\mathbb{R}$.
Definitions of functions of moderate decrease $M(\mathbb{R})$ and Schwartz space $S(\mathbb{R})$.
Properties and examples, $S(\mathbb{R}) \subseteq M(\mathbb{R})$
$K_\delta(x) = \delta^{-1/2} e^{-\pi x^2/\delta}, \ \delta > 0$ is a family of good kernels.
Properties of Fourier Transform on $\mathbb{R}$.