Exercises Compact Operators

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Notations:
X, Y : Banach Spaces, $\mathcal{H}$ Complex separable Hilbert space, $B(X,Y)$: Bounded linear operators from X to Y, $\mathcal{B}(\mathcal{H})$: Algebra of bounded linear operators on $\mathcal{H}$, $B_0(X,Y)$: Class of compact operators from X to Y. $\mathcal{B}_0(\mathcal{H})$: Class of compact operators on $\mathcal{H}$.

1. If $A \in B(X,Y)$ and either X or Y is finite dimensional, then show that A is compact operator.
2. If AB is compact, is it true that both A and B are compact ? Is it true that either A or B is compact ?
3. **True or False**: If T^2 is compact then T is compact.
4. If $A \in \mathcal{B}(\mathcal{H})$ is compact and invertible, then show that $\dim(\mathcal{H}) < \infty$.
5. If $A \in B_0(X,Y)$, then show that $\overline{ran(A)}$ is separable.
6. If $A \in B_0(X,Y)$, and $ran(A)$ is closed, then show that $ran(A)$ is finite dimensional.
7. Let $k(x,y)$ be a continuous function on $c([0,1] \times [0,1]$. Show that the operator $T : C[0,1] \to C[0,1]$ defined by $Tf(x) = \int_0^1 k(x,y)f(y)dy$ is compact
8. **True or False**: Strong limit of finite rank operators is cmpact.
9. Let $(a_n) \in c$ and $T : l^2 \to l^2$ be given by $T(b_n) = (a_n)(b_n)$. Prove that T is compact if and only if $(a_n) \in c_0$.
10. Show that no nonzero mupltiplication operator on $L^2[0,1]$ is compact.