AFS-II 2018, Assam University, Silchar, June 4-30, 2018

Lectures by individual resource persons:

1. Prof. A. B. Raha

Delivered four lectures and conducted two tutorials.

Lecture 1: At the outset, Prof. A. B. Raha introduced the motivation of Lebesgue measure to the participants that ultimately lead to the development of general measure theory. He defined ring, sigma-ring, field, sigma-field etc. and exemplify them. Some important properties of a sigma-field have also been discussed.

Lecture 2: Measurable functions have been defined and exemplified. Various types of combinations of measurable functions have been discussed in detail.

Lecture 3: Simple function is defined and exemplified. It has been proved that every measurable function can be expressed as a pointwise limit of a sequence of simple measurable functions.

Lecture 4: He defined measure and discussed various properties of it.

In the tutorial sessions, problems related to the topics discussed in my lectures were solved. Dr. Uday Shankar Chakraborty prepared the problem sheets and also helped me to conduct the tutorial classes.

2. Prof. A.R. Shastri

Lecture 1.
Soft copies of my pre-lecture notes were already distributed to the students. Prof. A.R. Shastri started with recalling some basic definitions and results from multi-variable calculus, proceeded to introduce the concept of parameterized curves.
Lecture 2.

He introduced the concept of curvature, proceeded to discuss the special case plane and space curves. He stated fundamental theorem on existence and uniqueness of space curves indicated a proof but could not complete the proof.

Dr. Naba Kants Sarma helped in preparing the tutorial sheets.

Lecture 3.

After introducing S. Mukhopadhyay's original version of Four vertex theorem, and requesting the students to read the proof from Pressley, he discussed Ossarmann's proof Knesser's version of it.

Lecture 4.

After stating the Isoperimetric inequality and requesting the students to read a proof of it from Pressley, he discussed some classical `proof' of it from the famous popular book `Stories of Maxima and Minima by Tikhomirov.

Parts of the tutorial hours were used to discuss Inverse and implicit function theorems and introducing the concept of a smooth manifold in the euclidean space.

2. Prof. Parvati. Shastri

1. Historical comments. Formal definition of ring, subring, ideal, quotient ring, examples. Subrings of C, ring of polynomials, of power series and ring of functions from a non empty set to a ring. Zero divisors, integral domain and fields. Characteristic, first definition. Homomorphism and isomorphism of rings.

2. Properties of Z and K[X]; Division algorithm in R[X], when the divisor is monic, application, principal ideal domains and examples of non-principal ideal domains, irreducible polynomials generate maximal ideal in K[X], K a field.

3. Fundamental homomorphism theorems, correspondence between ideals of a ring and ideals of quotient ring. Prime and maximal ideals, inverse image of prime ideal is prime ideal and of maximal ideal is maximal when homomorphism is surjective. Characterization of integral domains and fields using correspondence theorem.

4. Field of fractions of an integral domain, Prime rings and prime fields. Characteristic revisited. Finite fields must have p^n elements, for a unique prime number p and n $\in$ N.

5. Constructing a field in which a given non constant polynomial over a field has all the roots, sketch of proof of existence of finite fields having pn elements, as a field having the roots of $x^{p^n}-x$.

6. Maximal ideals of C[X_1, X_2, · · · , Xn], Hilbert’s nullstellensatz. Relation to algebraic geometry, solutions of family of polynomial equations, statement of Hilbert basis theorem for K[X_1, X_2, · · · , Xn], quotient of polynomial rings in several variables over an arbitrary field. Definition of Noetherian (as ring in which every ideal is finitely generated), more general statement of Hilbert basis tneorem: If R is Noetherian, so is R[X].

About tutorials: The tutorial sheets were distributed and some problems were discussed in full, with further comments.

About the students: Students participation was ok. As usual a few students were better than others, some are very poor, having problem with basics. Did not find any extra ordinary student. But some of them got excited and said they enjoyed the course; saw something new which they never had seen earlier, although the course was very elementary.

4. Prof. Alok Goswami:

Delivered four lectures and have conducted two tutorial classes in the program.

In the first lecture motivation for Lusin's theorem was discussed. In the second lecture, Prof. Alok Goswami proved Egoroff's theorem and with the help of this theorem one of the versions of Lusin's theorem was proved. Prof. Alok Goswami proved two more equivalent versions of Lusin's theorem in my third lecture. In my fourth lecture, Prof. Alok Goswami introduced Lp spaces and discussed their properties in detail. Lastly, the fourth version of Lusin's theorem viz. "for every complex valued measurable function with measure of the set where it does not vanish finite, and for every positive epsilon, there exists a continuous function g with compact support such that the measure of the set where f and g do not agree is less than epsilon".

In the tutorial sessions some problems related to Lusin's theorem, Egoroff's theorem and Lp spaces have been discussed. Both the tutorials were conducted jointly by Dr. Uday Shankar Chakraborty, Dr. A. M. Buhphang and me.

5. Dr. A. Tiken Singh

The following topics:

Surfaces in three dimensions, Examples of Surfaces, Quadric Surfaces, Tangents, Normals and Orientability, Applications of the Inverse Function Theorem, The first fundamental form , Lengths of Curves on Surfaces, Isometries of Surfaces , Conformal Mappings of Surfaces, Surface Area, Equiareal Maps and a Theorem of Archimedes Curvature of surfaces, The Second Fundamental Form, The Curvature of Curves on a Surface, The Normal and Principal Curvatures, Geometric Interpretation of Principal Curvatures, The Gaussian and Mean Curvatures, Geodesics.

6. Dr. Ritumoni Sarma

The following topics of Algebra.

Concept of Modules, Free modules, Diagonalizing integer matrices and its application to presentation of modules as generators and relations, Noetherian rings, Hilbert basis theorem, and structure of finite abelian groups. Followed ``Algebra" by Michael Artin.

7. Dr. K. V. Srikanth

The text followed was Elementary Differential Geometry by Andrew Pressley published by Springer.

June 26: Review of geodesic curvature, Derivation of geodesic curvature as rate of change of angle corrected by changes in frame, Statement and proof of Gauss--Bonnet Theorem for simple closed curves.

June 27: Statement and proof of Gauss--Bonnet Theorem for curvilinear polygons and geodesic polygons

June 27: Problem session in the afternoon based on above material.

June 29: Surfaces, Classification theorem of compact surfaces (without proof).

June 30: Triangulations, Euler Characteristic, Gauss Bonnet Theorem for compact surfaces without boundary.

June 30: Problem session in the afternoon based on above material.

8. Dr. Ardeline Mary Buhphang

Prescribed Book: Algebra, by Michael Artin

Was given to teach the topic of factorisation in rings in the Algebra course. The central idea of the course was to list the properties of the ring of integers and study about their validity in general integral domains. Dr. Ardeline Mary Buhphang gave four lectures and assisted in seven tutorials, four of which were in Algebra, two in Analysis and one in Topology during June 18-29, 2018.

The first class, Dr. Ardeline Mary Buhphang introduced the basic concepts on divisibility in integral domains. Some motivational remarks were noted as to why the concept was carried over from the ring Z of integers to an arbitrary integral domain. They discussed in detail the ring $Q[e, e^{\frac{1}{2}}, \dots, e^{\frac{1}{2^k}}, \dots]$ which provides an example of an integral domain which is not a factorisation domain. They also discussed about irreducible and prime elements and an example of a domain where there are irreducible elements which are not prime. In the second lecture, they discussed about Unique Factorisation Domains (U.F.D.'s) and their characterisations and properties and talked a little bit about GCD-domains which are generalisations of U.F.D.'s. In the third lecture, they talked about a class of domains called Bezout domains which lies between Principal ideal rings and GCD domains. In this lecture they also studied relations between P.I.D.'s and Euclidean domains. In the fourth lecture they confined our attention towards irreducible elements in Z[x] and Q[x]. We proved the Gauss'

Lemma and the Eisenstein's criterion and wrapped up the course with the result that polynomial rings of U.F.D.'s are again U.F.D.'s.

9. Dr. B. Subhash

6 hours of teaching and 4 hours of tutorial.

Topics covered:
Reisz Representation theorem: The complete proof was done in great detail, since the proof was quite lengthy the tutorial session was used for the same. Discussed the regularity properties of Borel measure. Introduced Lebesgue measure on Euclidean k dimensional space as the measure associated with Riemann integral on the space of continuous functions with compact support on Euclidean k dimensional space.

10. Dr. Uday Shankar Chakraborty

Delivered four lectures on Measure theory and have conducted two tutorials.
First lecture: Defined integral of a non-negative simple measurable function and have discussed some of its properties. Then integral of a nonnegative measurable function is defined.
Proved Monotone Convergence Theorem and discussed some its consequences including Fatou’s lemma thoroughly in my second lecture. he have also mentioned the importance of the concept “almost everywhere”.
Third lecture: integral of any measurable function was defined and properties related to it were discussed. The Dominated Convergence Theorem has been proved.
Last lecture, he had discussed convergence in measure and its relation with convergence pointwise almost everywhere. Finally he had restated Monotone convergence theorem in terms of convergence in measure and have proved accordingly.
Problems related to the topics have been discussed in both the tutorial classes. The first tutorial was conducted jointly by Dr. A. Tiken Singh, Dr. S. Mawiong and Dr. Uday Shankar Chakraborty and the second one was a joint venture of Dr. A. Tiken Singh, Mr. Naba Kanta Sarma and Dr. Uday Shankar Chakraborty.

11. Mr. Naba Kanta Sarma

1. In the first lecture, Mr. Naba Kanta Sarma began with two classical problems to motivate the lectures. he then introduced algebraic numbers and algebraic integers and prove equivalent conditions for an algebraic number to be an algebraic integer. He then showed that the set of all algebraic integers O forms an integral domain but not a factorisation domain.
2. In the second lecture, he introduced quadratic number fields and characterised them by establishing one to one correspondence with the square-free integers excluding 1. he then

introduced the ring O_K, of all algebraic integers in a given algebraic number field K and determined it explicitly when K is a quadratic number field. I then investigate several properties of O_K including non-uniqueness of factorisation and that O_K is a Dedekind domain.

3. In the third lecture, he computed the units of O_K for an imaginary quadratic field K. he also explained the real quadratic case by explicitly computing the units in $O_{Q(\sqrt{2})}$. he then proceed to determine all d < 0 for which $O_{Q(\sqrt{d})}$, d is a Euclidean domain. The case for d > 0 is also discussed at length.

4. In the last lecture, he investigated the ideals of O_K and developed the analogous concepts in the ideal

12. Dr. Sainkupar Mawiong

Did the tutoring for Analysis (Measure theory) and Topology (Differential Geometry). The students are very interactive and it seems that they really enjoyed the learning and the knowledge that they are getting from this school. The resource persons are very good and are well verse in their respective areas.

13. Dr. Pradip Debnath

Attended the AFS-II organised by Assam University, Silchar as tutor in Algebra. Dr. Pradip Debnath have conducted two tutorial classes jointly with Dr. Ritumoni Sarma and Mr. Naba Kanta Sarma during 11th June to 16th June, 2018.

Suggested topics for future meetings:

The participants have come with a wider range of suggestions for future programmes The subject on which they want future programmes are as follows:

Algebraic Number Theory, Differential geometry, Harmonic Analysis, Multivariable Calculus, Differential Equations, Fundamentals of Topology, Modular forms, Module Theory, Coding Theory, Cryptography etc.