NCM WORKSHOP: MODULAR FORMS AND GALOIS REPRESENTATIONS

Introduction to Rigid Analytic Geometry
Problem sheet (Tutor: Arvind Kumar)

(1) Let $|\cdot|$ be a non-Archimedean absolute value on a field K. Prove the following:
  (a) Let $a, b \in K$ satisfy $|a| \neq |b|$. Then
$$|a + b| = \max\{|a|, |b|\}.$$
  (b) Any triangle in K is isosceles.
  (c) Each point of a disc in K can serve as its center. Hence if an intersection of two disks is non-empty, they are concentric.

(2) Prove that T_n is a K-algebra.

(3) Let $|\cdot|$ be the Gauss norm on T_n. Then prove the following.
  (a) If $|f| = 1$, then there exists $c \in k$ with $|c| = 1$ such that $f + c$ is not a unit.
  (b)
$$\cap_{\mathfrak{m}} \mathfrak{m} = (0),$$
  where $\mathfrak{m}$ runs through the set of maximal ideals of T_n.

(4) Give an example of $f \in \mathbb{Q}_p\langle X\rangle$ such that $|f| > \sup_{x\in\mathbb{Z}_p} |f(x)|$.

(5) Let A, B be integral domains such that there is a finite injective morphism $A \hookrightarrow B$. Prove that A is a field if and only if B is a field.

(6) Let $f : A \to B$ be a morphism of K-affinoid algebras and let $\mathfrak{m} \subset B$ be a maximal ideal. Then prove that $f^{-1}(\mathfrak{m})$ is a maximal ideal in A.

(7) Prove that T_n is a K-Banach algebra (complete the proof given in the classroom).

(8) Prove that residue norm defined on an affinoid algebra is a norm.