NCM WORKSHOP: MODULAR FORMS AND GALOIS REPRESENTATIONS

Introduction to Rigid Analytic Geometry
Problem sheet (Tutor: Arvind Kumar)

(1) Let A be an affinoid K-algebra. Prove that the sets
$$D_f = \{x \in \mathrm{Sp}\ A : f(x) \neq 0\},\ \ f \in A$$
form a basis of the Zariski topology on Sp A.

(2) Show that for any affinoid K-space X the canonical topology is generated by the system of all sets
$$X(f) := \{x \in X : |f(x)| \leq 1\}$$
with f varying over A.

(3) Show that each Weierstrass domain in an affinoid K-space X is Laurent, and each Laurent domain in X is rational.

(4) Prove that a Laurent subdomain in an affinoid K-space X is an affinoid subdomain.

(5) Let $\phi : Y \to X$ be a morphism of affinoid K-spaces. Then prove that ϕ is continuous with respect to the strong Grothendieck topologies on X and Y.

(6) Show that the affinoid space Sp $\mathbb{Q}_p\langle T_1, \ldots, T_n\rangle$ (which is closed unit ball) is
  (a) disconnected with respect to the canonical topology,
  (b) connected with respect to the strong Grothendieck topology.

(7) If $A = A_1 \times A_2$ is a product of nonzero K-affinoid algebras, then show that Sp A_1 and Sp A_2 are affinoid subdomains of Sp A.

(8) Prove that Sp A is connected if and only if A has no nontrivial idempotents.

(9) Show that the Tate curve is a rigid analytic space (1-dim rigid analytic torus).