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1. Matrices of rotation are easy to write in $\mathbb{R}^2$ and $\mathbb{R}^3$, but it becomes difficult to express such matrices in higher dimensions. Therefore, to understand rotations in higher dimensions, Sir William Hamilton constructed Quaternions and Quaternion multiplication (An alternative operation on $\mathbb{R}^4$ resembling complex multiplication).
2. Hamilton's Discovery of Quaternions (Article).
3. Definition of a Quaternion, Quaternion addition and multiplication in $\mathbb{R}^3$.
4. Verifying properties of Associativity, Distributivity, Conjugation, Length rule etc. over Quaternion addition and multiplication in $\mathbb{R}^3$.
5. Extension of Quaternions to $\mathbb{R}^4$. Definition of Quaternion multiplication * in $\mathbb{R}^4$.
6. Quaternion Algebra: Space $\mathbb{R}^4$ equipped with * is called Quaternion Algebra.
7. Description of all Quaternions of unit length.
8. Sums of squares over fields - Brahmagupta (628 A.D.) – Product of sums of two squares is again a sum of squares. i.e. $(a^2+b^2)(c^2+d^2) = (ac-bd)^2+(ad+bc)^2$. The statement is equivalent to multiplicativity of complex norms : $\|z_1 z_2\| = \|z_1\|\ \|z_2\|$
9. Example to show that Product of sums of three squares may not be equal to sums of 3 squares.
10. Lagrange (1770): Product of sums of four squares is again a sum of four squares.
11. Hurwitz Theorem (Multiplication of norms happens in dimension 2, 4, 8)
12. Fibonacci sequence and other similar iterative processes.
13. If F_n denotes the n^{th} element of Fibonacci sequence, then: $cot^{-1}(F_3) + cot^{-1}(F_5) + cot^{-1}(F_7) + \cdots = \frac{\pi}{4}$.
14. Finding a formula for F_n in matrix form.
15. Lucas sequence and finding a formula for L_n in matrix form.
16. From Linear algebra to Google: Google's ranking algorithm, Page rank of websites.
17. Application of linear Algebra to Facebook: Number of mutual friends problem- If A is adjacency matrix of a graph, then entries of A^2, A^3 tell us the number of paths from vertex v_i to v_j of degrees 2 and 3 respectively.
18. Least squares approximation problem: To find line of best fit into given data using linear algebra techniques.