## Prof. Shanta Laishram

1. Zorn's Lemma, Well ordering principle.
2. Let $a, b$ be non-zero integers. Then $\gcd(a, b) = am + bn$, for some $m, n \in \mathbb{N}$.
3. Ring, Zero divisor, Integral domain, Skew field (Division ring), Field, Polynomial ring.
4. Group Ring $R[G]$ = Set of finite linear combinations of elements of $G$ with coefficients in $R$. $R[G]$ is a ring.
5. Quaternion Ring : $H = \{a + bi + cj + dk : a, b, c, d \in \mathbb{Z} ;\ i^2 = j^2 = k^2 = -1 ;\ i.j = k,\ j.k = i, k.i = j\}$. $H$ is a skew field in $\mathbb{R}^4$ .
6. Characteristic of a ring, Ring Homomorphism, Isomorphism, Kernel, Ideal, Quotient Ring, Ideal generated by a nonempty subset X of R.
7. Unit group of a ring $R$ is $R^* = \{a \in R : a \text{ is a unit in } R\}$.
8. Factor Theorem for ideals in a ring.
9. First, Second and Third Isomorphism theorems.
10. Direct Product of rings.
11. The Chinese Remainder Theorem and its applications.
12. Product of ideals.
13. Extension fields : Let $K$ be a field which is a finite extension of $\mathbb{Q}$. i.e., $K$ is a finite dimensional vector space over $\mathbb{Q}$. Then the dimension of $K$ over $\mathbb{Q}$ is denoted by [K : $\mathbb{Q}$], and is called the degree of extension.
14. Algebraic number : $\alpha \in \mathbb{C}$ is called an algebraic number if there exists a polynomial $f(x) \in \mathbb{Z}[x]$, such that $f(\alpha) = 0$.
15. Transcendental number.
16. Minimal polynomial: If $\alpha$ is an algebraic number, then the monic polynomial of least positive degree over $\mathbb{Q}$ for which $\alpha$ is a root, is called the minimal polynomial of $\alpha$ , denoted by min($\alpha$).

    min($\alpha$) is irreducible.
17. Theorem: If $\alpha \in K$ (A number field or an extension field), then $\alpha$ is an algebraic integer if and only if $\mathbb{Z}[\alpha]$ is a finitely generated abelian group.
18. Let $K$ be a quadratic field. Then $K = \mathbb{Q}[D]$ for a unique square free integer D.
19. Pell's equation, Ramanujan Nagell's equation.
20. Euclidean map, Euclidean domain, norm, Euclidean quadratic field, Primes and Irreducibles in an Integral domain.
21. Unique factorization domain (UFD), Principle Ideal Domain (PID).
22. Applications to number theory.